Using Mathematical Tables

A mathematical table serves to provide the value of some function $F(x)$ of a variable x, for various values of x. Mathematical tables are published both within textbooks such as this one, and also as collections of tables in stand-alone publications with titles like "Mathematical Tables" or "Statistical Tables." The resolution provided by tables, in terms of the number of different values of x covered, affects both their size and cost. Increasing the number of x values inevitably requires more pages for a given table, and hence increases both size and publication cost.

Figure A4.1 shows part of the error function table that is presented in complete form as Table 4.1 in Chapter 4. This provides values of the function $F(z)$ for various values of z, where $F(z) = \int_{-\infty}^{z} \frac{1}{\sigma\sqrt{2\pi}} e^{(-z^2/2)} dz$.

In the table in Figure A4.1, the left-hand column gives various values of z with one figure after the decimal point from 1.0 to 1.9. The number at the top of the columns to the right gives the second figure after the decimal point for the value of z, from 0.00 to 0.09. The numbers in the rest of the table give the value of $F(z)$ corresponding to the value of z given at the left-hand end of each row and the "second figure after the decimal point" shown at the top. This will become clearer with some examples.

	F(z)									
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8906	0.8925	0.8943	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9648	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

Figure A4.1
Part of an error function table.

If we want the value of $F(z)$ corresponding to $z = 1.40$, we look down the left-hand column until we find "1.4" and we then look along this row to the column that has "0.00" at the top (column 2). This gives a value of $F(z) = 0.9192$.

If we want the value of $F(z)$ corresponding to $z = 1.78$, we look down the left-hand column until we find "1.7" and we then look along this row to the column that has "0.08" at the top (column 10). This gives a value of $F(z) = 0.9625$.

The relevant numbers for both these examples are highlighted by shading in Figure A4.1.

Interpolation

It frequently happens that the value of a parameter is expressed to a greater number of figures after the decimal point than is provided for in a mathematical table. For example, suppose we want to find the value of $F(z)$ for $z = 1.783$. This value of z has three figures after the decimal point, but the table only provides for values of z with two figures after the decimal point. We can find a value of $F(z) = 0.9625$ corresponding to $z = 1.78$ from the table and we can also find a value of $F(z) = 0.9633$ corresponding to $z = 1.79$ from the table. The required value of $F(z)$ corresponding to $z = 1.783$ lies between these two values of $F(z)$. We need to use *interpolation* to determine the exact value of $F(z)$. The process of interpolation is illustrated in Figure A4.2.

If we had a graphical representation of the relationship between $F(z)$ and z, it would be a curve, and we could show the part of the curve corresponding to values of z between 1.78 and 1.79 as in Figure A4.2. To find the value of $F(z)$ corresponding to $z = 1.783$, we would measure 3/10 of the distance along the z-axis between $z = 1.78$ and $z = 1.79$ and then draw a vertical line up to the curve of $F(z)$ against z (shown as a vertical dashed line in the figure). Then, drawing a horizontal line from the point of intersection with the curve

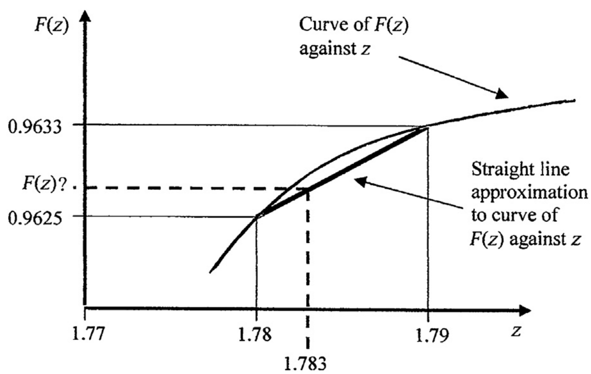

Figure A4.2
Illustrating the process of interpolation in a data table.

along to the $F(z)$-axis would give the correct value of $F(z)$ for $z = 1.783$ at the point where this horizontal line cuts the $F(z)$ axis.

However, in practice, we do not have a graph of $F(z)$ against z available. All we have are the $F(z)$ data points for $z = 1.78$ and for $z = 1.79$ given in a table. What interpolation does is to effectively draw an imaginary straight line between the data points at $z = 1.78$ and $z = 1.79$ (which is an approximation to the actual curve between these two data points). The vertical line up from the value of $z = 1.783$ is drawn as far as this straight line and then the horizontal line from the point of intersection to the $F(z)$ axis (shown as a horizontal dashed line in the figure) defines the value of $F(z)$ corresponding to $z = 1.783$. As can be seen in this example, this is an approximation to the correct value of $F(z)$, but it is the best we can get when we have a value of z that is between the values given in the table.

The process of calculating the value of $F(z)$ for a value $z = 1.783$ can therefore be summarized as follows.

Since the $F(z)$ value for $z = 1.783$ is 3/10 of the distance between $z = 1.78$ and $z = 1.79$, the value of $F(z)$ for $z = 1.783$ is 3/10 of the distance between the values of $F(z)$ corresponding to $z = 1.78$ and $z = 1.79$. Hence, $F(1.783)$ is calculated as:

$$F(1.783) = F(1.780) + 0.3[F(1.79) - F(1.78)] = 0.9625 + 0.3[0.9633 - 0.9625] = 0.9627$$

The interpolation procedure is exactly the same if we have four or more figures after the decimal point for the value of z (although the change in the value of $F(z)$ will often be negligible for such a very small change in the value of z). The correct value of $F(z)$ for $z = 1.7836$ is 36/100 of the distance between the values of $F(z)$ corresponding to $z = 1.78$ and $z = 1.79$.

Hence, $F(1.7836) = F(1.780) + 0.36[F(1.79) - F(1.78)] = 0.9625 + 0.36[0.9633 - 0.9625] = 0.9628$.